ADVENTURES BEYOND THE FRINGE
Radio Interferometry in the X Band

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ABSTRACT

We present a report on the activities concerning the radioastronomy technique known as interferometry. We first discuss coordinate systems, coordinate system conversions and precession. We prelude our technical findings with an exhaustive discussion on radio interferometry theory, in which we cover interference, point source response, and notes on our technical equipment. The main body of the report consists of using interferometry to measure accurate declinations of point sources and accurate angular diameters of extended sources. We cover both the theory and practice of these two fundamental uses of interferometry. Finally, we present brute force and non-linear least squares techniques. We studied the sun and moon as our extended sources and the Crab Nebula as a point source with UC Berkeley's undergraduate interferometer.

[1]Document created February 13, 2007

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1. INTRODUCTION

Radio astronomers use the technique known as interferometry to create and study a fringe pattern caused by the interference of a source's radio emissions between multiple radio telescopes. This technique combines multiple radio telescopes into a single detector, allowing us to achieve angular resolutions much higher than a single radio telescope, since a single dish is limited by diffraction according to $\frac{\lambda}{\mathcal{D}}$ (Where $\mathcal{D}$ is the diameter of the antenna dish), while we effectively create an aperture as big as the baseline length between the telescopes. Radio astronomy is limited by the very long wavelengths we observe at, which cause very low angular resolutions, and this method allows us to vastly improve this limit, since we can effectively create arbitrary-length baselines to have incredible resolving power (For example, two radio telescopes separated by the diameter of the earth), and we can boost sensitivity by connecting incredibly sensitive telescopes such as Arecibo in Puerto Rico and the Very Large Array in New Mexico.[2]

2. COORDINATE SYSTEMS

A study of the celestial bodies demands, first and foremost, a systematic approach to labeling the positions of the celestial and stellar objects. Several coordinate systems are in use today, of which the Horizontal coordinate system and the Equatorial coordinate system are of special importance to us.

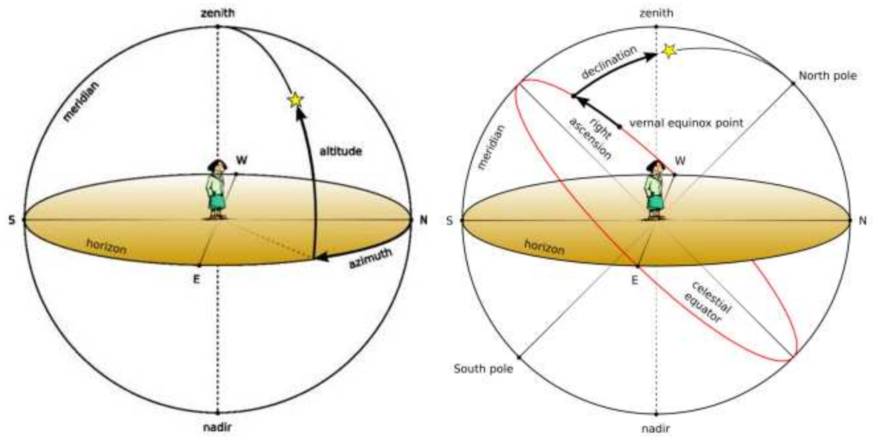

Fig. 1.— The Horizontal and Equatorial Coordinate System

The Horizontal coordinate system uses the (az, alt) coordinates to describe any position in the sky relative to the observer's position on the earth. This system divides the sky into a plane tangent to the observer's position on the earth, with altitude being the angle above or below this plane, and azimuth the compass angle from North. The convenience of this system comes from its observer-centered nature. For example, any object below the horizon has a negative altitude, and is thus not visible. Our radio telescopes are also on an *altazimuth* mount, which means that it has two perpendicular directions of movement directly corresponding to the (az, alt) coordinates.

The Equatorial coordinate system is independent of observer position and fixed to the stars[3], not the earth. It uses the the (α, δ) coordinates (known as Right Ascension and Declination respectively) to describe the

[2] Which is why my radio professor says your optical professor needs to get his act together. We radio astronomers have great sensitivity and resolving power, and we are diffraction-limited, versus most optical telescopes, that are seeing-limited by atmospheric effects.

[3] Except for precession effects, which causes a slow drift of objects against this coordinate system, and needs to be accounted for on a scale of years to dozens of years.

position of an object in the sky relative to the celestial equator and celestial poles, which are the projections of the earth's equator and poles. The declination measures the angle above or below the celestial equator, and the right ascension measures the angle east of the Vernal Equinox.[4]

2.1. Accounting for Precession

Since we use the Celestial Coordinate System that defines Declination and Right Ascension against the sky, we must account for precession of stars against the celestial "background" that occurs over a multitude of years. To do this we use a set of IDL instructions that takes a known D, RA value, convert it to degrees, and calculate the precession over a known time interval:

```
precess, ten(h, m, s)*15, ten(deg, min), thenyear, nowyear, /PRINT
```

This IDL command will print out the new, precessed (α, δ) values.

2.2. Converting between Equatorial and Horizontal coordinate systems

The position of celestial objects are normally given in the Equatorial coordinates, since this is observer and (relatively) time independent. Our telescopes are configured in the Horizontal coordinate system, so we need to convert between these coordinate systems.

There is two rotations involved in converting (α, δ) to (az, alt). First we rotate around the equatorial poles by an angle equal to the local sidereal time. This changes the right ascension coordinate to hour angle, which is the angular distance between the object and the local meridian[5] Secondly we rotate the celestial equator to change the hour angle and declination into azimuth and altitude respectively.

Thus, the step-by-step procedure we use to convert from the Equatorial to Horizontal coordinate system is:

- Convert the Declination to Hour Angle, noticing that the hour angle goes in the opposite direction as the right ascension:

$$ha = LST - \alpha \tag{1}$$

- Reduce the spherical coordinates of (α, δ) to a vector of rectangular coordinates:

$$\mathcal{X} = \begin{bmatrix} \cos\delta \cos ha \\ \cos\delta \sin ha \\ \sin\delta \end{bmatrix} \tag{2}$$

- Create a rotation matrix that rotates and flips the rectangular coordinates to convert (α, ha) to (az, alt), where ϕ is the station's terrestrial latitude:

$$\mathcal{R} = \begin{bmatrix} -\sin\phi & 0 & \cos\phi \\ 0 & -1 & 0 \\ \cos\phi & 0 & \sin\phi \end{bmatrix} \tag{3}$$

- Apply this rotation matrix to the rectangular (α, ha) vector to generate the vector in rectangular coordinates that represents the point in the sky according to the Horizontal coordinate system.

$$\mathcal{X}' = \mathcal{R}\mathcal{X} \tag{4}$$

[4] *Vernal Equinox:* The point where the sun passes from South to North for an observer in the Northern Hemisphere. In other words, the place where the Great Circle of the Celestial Equator and the Great Circle of the Ecliptic (the sun's path) intersect each other.

[5] *Meridian:* The imaginary great circle that passes through the celestial poles and the observer's zenith.

- Convert the rectangular coordinates to spherical coordinates:

$$azimuth = \arctan\left(\frac{\mathcal{X}'[1]}{\mathcal{X}'[0]}\right) \tag{5a}$$

$$altitude = \arcsin\left(\mathcal{X}'[2]\right) \tag{5b}$$

3. INTERFEROMETRY

3.1. Monochromatic Point Sources

A monochromatic point source in the sky emits electromagnetic waves that we observe using radio telescopes. We can easily model the wave from a monochromatic source as the intensity of the electromagnetic wave. Incidentally, this is also what we would expect to measure if we were using a single detector.

$$E(t) = \cos(2\pi\nu_0 t) \tag{6}$$

The electromagnetic wave that we observe from the source is for all practical purposes an infinite plane wave with all points moving parallel to each other when it reaches our detector.[6] Different observers will detect the wave at different point in time. Since we assume that the plane wave approaches the earth uniformly, the time difference will depend only on where the observers are located. Figure 2 and Equation 7 demonstrates the simplest 2-dimensional case of this path length difference for two observers separated by distance $\mathcal{D}$. This corresponds to an interferometer with a north-south baseline observing a source moving along the meridian.[7]

$$\delta P(h) = \mathcal{D}\sin(h) \tag{7}$$

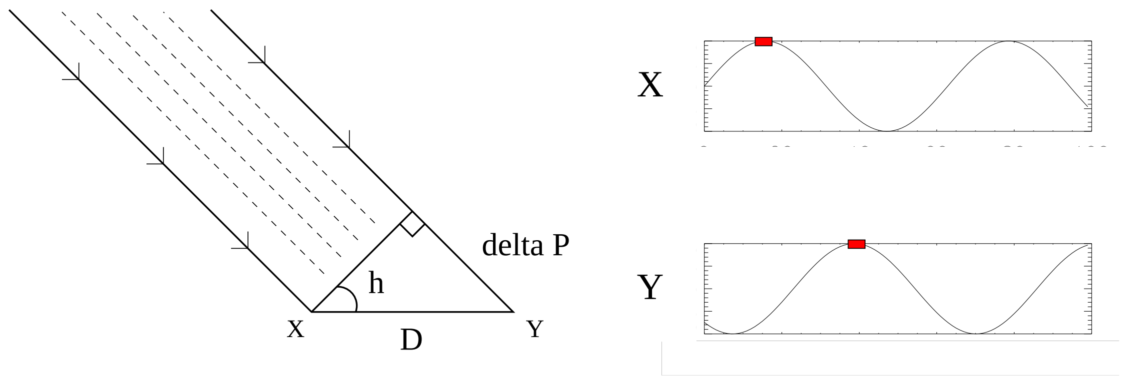

Fig. 2.— Path Length Difference (δP) between and observed $E(t)$ of two telescopes observing the same source

3.2. Two-Telescope observation of Point Sources

Interferometry exploits the wave nature of light by mixing the signals of multiple detectors to form an interference fringe. Consider a single plane of a plane wave falling in on our detectors. The relative path length difference between the detectors means that a specific plane of the incoming plane wave reaches each detector at a slightly different time. This time difference offsets the different waves by different amounts. This offset logically gives rise to phase differences between the waves. The fringe is caused by destructive and constructive interference between the waves in the mixing process. The incredibly powerful aspect of interferometry is its dependence on this *relative path length* between detectors and the related *relative phase difference* between each detected signal. The differences in phase is related only to the path length

[6]From the *geometric optics* viewpoint, this means that all the rays from the source approach parallel to each other.

[7]Please note that this is a 2D simplification of the actual case! Rarely do we have objects moving perfectly along the meridian.

differences and the *wavelength* of observation. The total distance the waves travel - on the order of parsecs - is inconsequential to the path length difference between the detectors.[8] The interferometer fringe is not a static interference pattern. Sources moves across the sky as the earth rotates, and the relative path length δP changes as the angle h changes. Figure 3 gives a graphical demonstration of this.

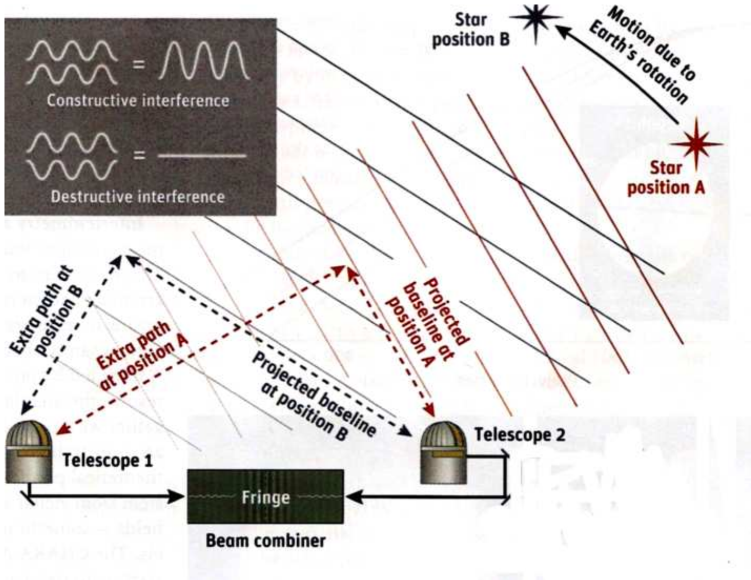

Fig. 3.— Change in observed fringe pattern, relative to change in apparent baseline, as the source moves through sky. (Source: *Sky and Telescope* Feb 2007)

Our two dimensional model can be generalized to a much more useful three dimensional case using simple geometry. For an east-west baseline, the distance $\mathcal{D}$ between the two detectors is a function of declination. For simplicity, we assume we are on the equator (the math remains general, but visualization is significantly easier!) and we have an east-west baseline. The celestial equator intersects our local zenith, and a declination of zero implies that the source lies on this great circle. As long as the declination remains zero, our 2D simplification holds perfectly, with angle h becoming the hour angle. On the other extreme, if the object lies on the horizon, i.e. is has a declination of 90°, a change in hour angle does not change the path length at all, since we are rotating with respect to an axis that is parallel to the two beams of the telescope. It is easy to infer the geometry from here on. Thus, the time difference[9] between the two detectors is directly related to the sine of the hour angle and the cosine of the declination. The constant factor that converts this to a time difference is the distance between the detectors divided by the wavelength we are observing.

$$\tau_g(h) = \left(\frac{\mathcal{B}_y}{c}\cos\delta\right)\sin h \tag{8}$$

It is important to remember that our beautiful generalization only holds until the incoming waves hit the detector. The two waves travel through a long path of electronics that enables the mixing and sampling process, and this adds a constant path length difference since the path of each wave differs from the other. We

[8]We are, of course, limited by the intensity drop off as the path length increases, but interferometry is equally effective for two sources with the same apparent magnitude and wildly varying distances.

[9]we call this the "geometric time difference" τ_g since it only depends on the geometry of the observation

could theoretically try to minimize this difference in path length through our electronics, but each tiny cable cut that introduces a couple of micrometers difference will introduce a detectable path length difference. The sensitivity of the interferometer can be seen as a boon, but since we can easily take this constant difference into account mathematically, all we need to assure is that the path length difference through the electronics does not change over the course of a single observation. Thus, our complete formula for the measured time difference between the observation of the same source by two detectors is:

$$\tau_{net}(h) \quad = \quad \left(\frac{\mathcal{B}_y}{c}\cos\delta\right)\sin h \quad + \quad \tau_c \tag{9}$$

3.2.1. Interferometer Response to a Point Source

We can examine the interferometer response to a point source now that we have an expression for the time difference between the signals from two detectors. If we combine equations 9 and 6 we find the equations for the signal detected by each dish:

$$E_1(t) = \cos\left(2\pi\nu_0 t\right) \tag{10a}$$

$$E_2(t) = \cos\left(2\pi\nu_0[t + \tau_{net}]\right) \tag{10b}$$

The interferometer we used is a multiplying interferometer. The mixing process multiplies the two signals together, producing the interferometer fringe output $F(t) = E_1(t)E_2(t)$

$$F(t) \;=\; \cos\left(2\pi\nu_0 t\right)\cos\left(2\pi\nu_0[t + \tau_{net}]\right) \tag{11}$$

This expression, albeit mathematically correct, is difficult to work with in fitting curves to data. It contains the product of two non-linear functions with a dependence on each other. It would be preferable to have this as a sum of unknowns, each term with only one unknown. Through the use of two trig identities, this formula can be converted into the form $F(t) = As_m + Bt_m$. We will discuss the importance of this form in section 4.2. First we expand it into two cosines[10], one dependent on the sum of the arguments, the other dependent on the difference:

$$F(t) = \frac{1}{2}\left[\cos(2\pi\nu\,\tau_{net}) + \cos(4\pi\nu\,t + 2\pi\nu\,\tau_{net})\right]$$

Notice that the first term varies at a very high rate. We are not interested in this term since it does not contain any information that is not already present in the other term, and solely depends on the time, not the time difference. We drop this term, and we will filter this out using a low-pass filter in our electronics.

Note: $\tau_{net}(h)$ can be expressed as $\tau_{net}(t)$ since the angle h is the hour angle ha. ha changes with the Local Sidereal Time, expressed in hours. Thus, we can make the substitution $h = t$.

$$F(t) = \cos(2\pi\nu\,\tau_{net}) = \cos(2\pi\nu\,[\tau_g(t) + \tau_c]) \tag{12}$$

[10] $\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

Expanding[11] this term with the following substitutions

$$\nu\left(\frac{\mathcal{B}_y}{c}\right)=\left(\frac{\mathcal{B}_y}{\lambda}\right)$$
$$A=\cos\left(2\pi\nu\,\tau_c\right)$$
$$B=\sin\left(2\pi\nu\,\tau_c\right)$$

produces the **Interferometer Response:**

$$\boldsymbol{F(t)=A\cos\left[2\pi\left(\frac{\mathcal{B}_y}{\lambda}\cos\delta\right)\sin h\right]-B\sin\left[2\pi\left(\frac{\mathcal{B}_y}{\lambda}\cos\delta\right)\sin h\right]} \tag{13}$$

3.2.2. Local Fringe Frequency

The argument of the sine and cosine function in the interferometer response can be seen as a constant times time. The declination for a point source does not change except for precession, which is on the order of years, not days, so we do not need to take it into account for our short measurement cycles. The hour angle is directly proportional to time, so we call the hour angle "time"[12]. We want to develop the concept of a *local fringe frequency*, which we can do from these observations. The argument is the product of a constant and $\sin(h)$. This product makes the sine and cosine functions oscillate by a frequency dependent on the current hour angle. Through Taylor expansion, we can derive the result for the local fringe frequency in terms of cycles per radian in the sky, which is as follows:

$$f_f=\left(\frac{\mathcal{B}_y}{\lambda}\cos\delta\right)\cos(h_s) \tag{14}$$

3.3. Our Interferometer

We used two 1 meter radio dishes with a baseline length $\mathcal{B}_y = 9.05m$ at an observing frequency of $\nu = 10.698GHz$, $\lambda = 0.028\,m$ in the X-band. This gives us an angular resolution equal to:

$$\theta=\frac{\lambda}{\mathcal{B}_y}=\frac{0.028\,m}{9.05\,m}=10.6\,' \tag{15}$$

Thus, any object with an angular size less that 10.6 arc minutes will appear as a point source to our interferometer. This means that just about every object in the sky is a point source!

[11] $\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$

[12] The hour angle $ha = lst - ra$, see section 2.2

4. ACCURATE MEASUREMENT OF THE DECLINATION OF A POINT SOURCE

4.1. Background & Observations

If we look back at equation 13 we notice that the interferometer response to a point source is a function of position in the sky (assuming you know your baseline and observation wavelength). We can exploit this fact to use interferometry to accurately measure positions of point sources.

We tracked the Crab Nebula (M1) over the course of a night as it made its journey across our night sky. The Crab Nebula is an extremely bright radio source. It is powered by the Crab pulsar, a ˜1000 year old supernova remnant about 1kpc distant. By tracking it over the course of a full transition of the sky, we record the fringe frequency as it changes by quite a lot. This data will allow us to solve for the unknown position values we look for.

4.2. Analysis: "Brute Force" iterative least squares fitting

We are interested in finding very accurate declination values for M1. If we assume we know the hour angle with a high degree of precision, we can use the technique known as "Least Squares fitting" to find the value of the true declination. How do we do this? After observing M1 over the course of one night, we have rough data against which we can try to fit our generic mathematical model we developed in section 3.2.1. We want to fit equation 13 to our data. There are two unknowns (A and B), each dependent on the path length differences inside our electronics. We have two coefficients, each dependent on the hour angle. We have our data which we are fitting against, keeping in mind that the data has some noise and other uncertainties. Before we get lost in the math and intricacies, let's evaluate what we are trying to do. We have a theoretical interferometry response to what we are observing. We also have the data we collected by observing the Crab Nebula. Least Squares fitting solves the unknowns in our theoretical model, which we use to find the closest match of the theoretical model to our data. Linear Least Squares fitting normally takes on the form $As_m + Bt_m + Cu_m + D_v m = y_m$, where we are solving for A through D.

Before we begin fitting curves to our data, we first prepare our data to ensure the best possible fit. Figure 4 shows the data we collected as we tracked the Crab Nebula. I normalized this data by fitting a 10 degree polynomial to it (showed in figure 4. The difference is the original signal situated around 0.

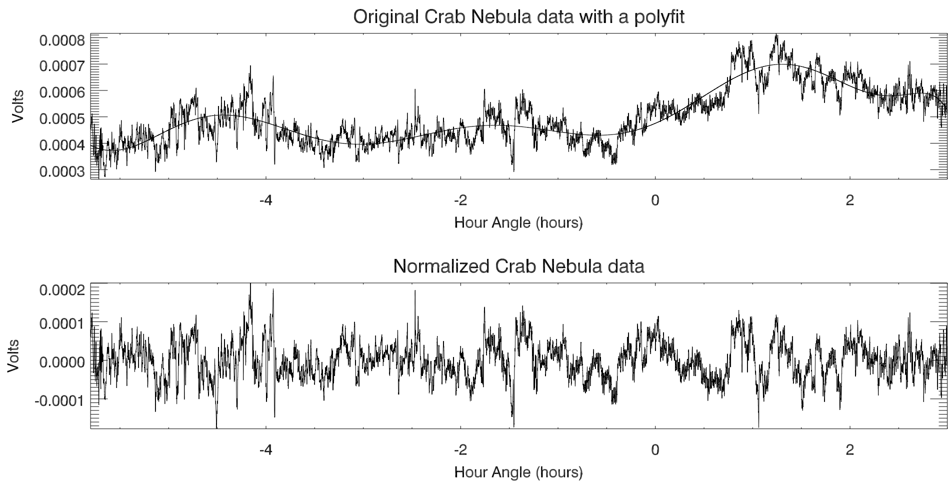

Fig. 4.— Crab Nebula Raw, Poly-fitted and Normalized Data

We approach this problem in the "Brute Force" way. We can solve for the two unknowns A and B using least squares - but ultimately we're interested in the declination value. The solution is simple - we reverse-engineer! Since we pointed our telescope at the source, we have a rough idea of where it is in the sky. We take this as our starting guess value, and do multiple fits using least squares around this known value. Since error calculation is a very important and integral part of the Least Squares process, we end up with a graph of guessed declination values versus calculated sample variance s^2. It is obvious that the best fit will be the one with the smallest sample variance, and once we have this value, we know the declination!

I attacked this problem by trying to match as much data as possible. The strange result of my fits can be seen in figure 5. Since any object can only have one declination value, the *two* dips in sample variance points to a problem somewhere.

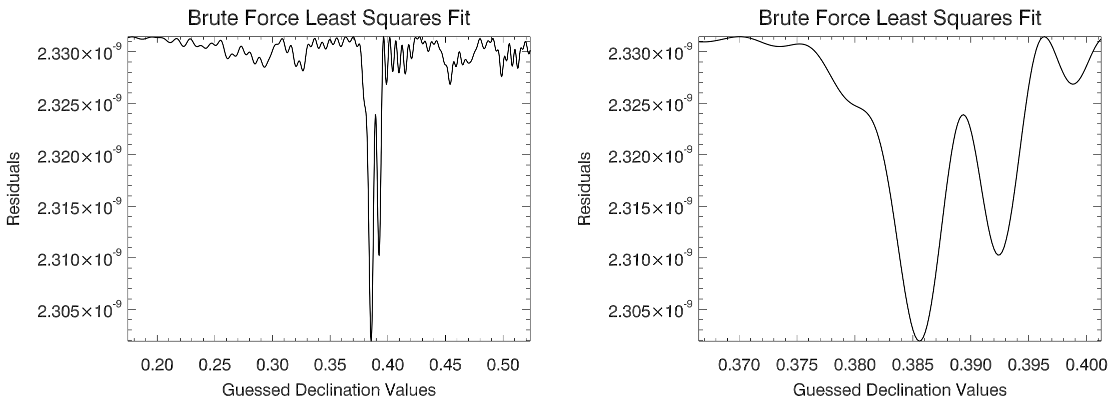

Fig. 5.— The sample variance of our least squares fit plotted against the guessed declination.

If any of our assumed constants change over the course of an observation, our fits will be thrown off. The constant that is most easily influenced by external circumstances is our observation wavelength λ, since it is determined by oscillators in our equipment. Someone hitting a cable or bumping into the table, sudden heat changes and a plethora of other events can alter λ. The easiest way to still retrieve useful information from our data is to divide it into chunks and work with each chunk separately. Through inspection, we should quickly be able to tell if this is indeed where the problem lies or not.

After dividing our data into several chunks, each of about 3 hours wide, I fitted against each of these chunks. My suspicions were confirmed when the second dip did not show up until i reached hour angles in excess of 0.5 hours. Truncating my data to the section $h = 3.8$ to $h = 0.3$ allowed me to match my theoretical value to the interferometer fringe response, as figure 6 and 7 shows.

We used the variances in derived coefficients to find the sample variance, which allowed us to get a quantative measure of the errors in our fit. Another important and helpful indicator of the accuracy of our fit is the *covariance* - the *degree to which the uncertainty in one derived coefficient affects the uncertainty in another derived coefficient.* Through some more matrix manipulation in IDL we found the covariance matrix at our best fit to be the following:

$$\begin{bmatrix} 1.0 & 0.00081224966 \\ 0.00081224966 & 1.0 \end{bmatrix} \tag{16}$$

It is very clear that there is very little dependence between the uncertainties in variables. This is another indicator that we do have an excellent fit!

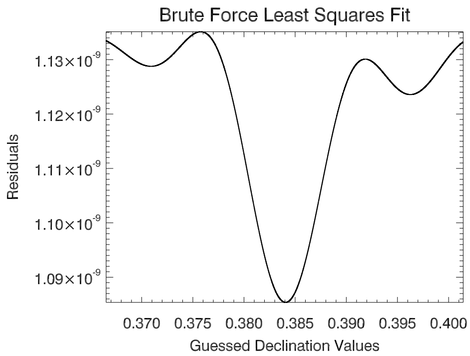

Fig. 6.— Least Squares fit, guessing from to with a truncated data set from ha=-3.8 to ha=0.3

4.3. Results

We managed to find coefficients that fitted our theoretical response to our measured response even though there was some problems with our data. We successfully used Least Squares fitting and error analysis to find an accurate declination value of the Crab Nebula, namely $20° \; 0' \; 18.95''$ (0.38406436 rad). This was derived by XS guessing in $\frac{1}{30000}$ intervals over the range $(20°, 22°)$. My covariance matrix indicated that the uncertainties in each variable had little dependence on each other, and my sample variance was at its smallest at this point. I plotted the predicted Interferometer response against the data in figure 7 and examined the shape of the two curves. The mathematical error indicators were confirmed by the excellent match in shapes of the two curves. I took the liberty of checking my derived declination value against the accepted position of the Crab Nebula and found my value within 10 arc-seconds.

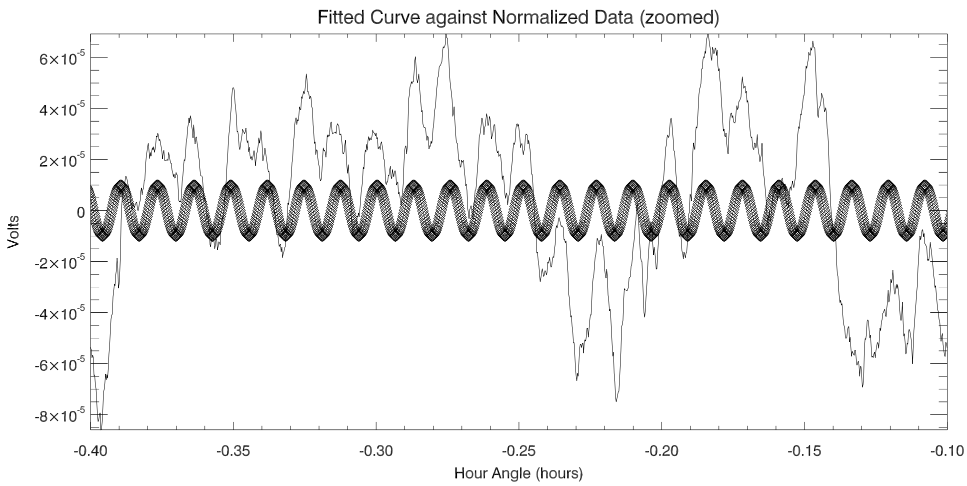

Fig. 7.— Least Squares predicted curve against normalized data (zoomed).

5. MAKING MAPS OF THE SKY - INTERFEROMETRY OF EXTENDED SOURCES

5.1. Interferometry Response to an extended source

In section 3.2.1 we examined the interferometry response to a point source. As the object moves across the sky[13], the apparent baseline changes. A different viewpoint of this system is looking at the interferometer projecting a fringe pattern against the sky. As the sky rotates, the source moves through this fringe pattern to give the fringe response $\mathcal{R}(t)$ where t is equal to the hour angle[14]. The response for a point source is given by equation 13. For an extended source[15], we need to integrate this point source response over the extent of the source. At any point in time, the fringe pattern $\mathcal{F}(h)$ covers the source and the interferometer response $\mathcal{R}(h)$ is the integral of the fringe pattern times the source intensity distribution.

Some clever mathematical manipulation brings us to the final usable form of the interferometer response to an extended source:

$$\mathcal{R}(h) = [A\cos[\alpha(h)] + B\sin[\alpha(h)]] \times \int I(\Delta h)\cos[\beta(h,\Delta h)]d\Delta h \tag{17}$$

Notice that this is the point source value modulated by another function! We rewrite this as:

$$\mathcal{R}(t) = \mathcal{F}(t) \times MF_{theory} \tag{18}$$

$$MF_{theory} = \int I(\Delta h)\cos(2\pi f_f \Delta h)d\Delta h \tag{19}$$

The modulation function contains crucial information about the source structure. Since the modulation function is generally the Fourier transform of the source intensity distribution, we can calculate the modulation function and find interesting properties about the source.

5.2. The Modulation Function & Calculating Diameter

We will be observing circular sources in the sky - the sun and the moon. The intensity function of a round source is the following:

$$I(\Delta h) = \frac{(R^2 - \Delta h^2)^{1/2}}{R} \tag{20}$$

We incorporate this into equation 19 to find the modulation function for the sun or moon:

$$MF_{theory} = \frac{1}{R}\int_{-R}^{R}(R^2 - \Delta h^2)^{1/2}\cos(2\pi f_f \Delta h)d\Delta h \tag{21}$$

What do we do with this equation? It gives us the function by which the point source interferometer response is modulated when we study an extended source. It is a function of $\boldsymbol{f_f R}$, thus it depends on the fringe frequency given by equation 14 and the **diameter of the source!**. How can we use our data and this theoretical function to find this diameter? The aspect of this formula that we can use to our advantage is its *zero points*. For a declination value $\delta = 0$ and a known hour angle value $h! = 0$, the only factor that can make the interferometer response equal to zero is the modulation function. (In other words, we have to assume that the declination is small and fairly constant for this method to work). We numerically evaluate

[13]In reality, the earth turns under the sky, and the sky rotates above us.

[14]We can do this since we study the sun and moon, which are always close to zero declination and we have an east-west baseline.

[15]Keep in mind that "extended" is defined by our array's angular resolution $\frac{\lambda}{D}$

this integral to create the plot of the modulation function. Notice the axes! Y is the relative amplitude of the modulation function versus a range of $f_f \times R$ value on the X-axis.

If we have the zero points given by this theoretical model (and after numerically evaluating the integral, finding the zero points simply means reading it off the plot) and we have the exact points where our interferometer recorded an amplitude of zero, we can trivially calculate the R value - the diameter of our extended source!

Why does this happen? The fringe frequency changes across the sky - thus, as the source moves through the sky, the number of fringes across it will vary. When an integer number of fringe periods cover the source, integrating the point source response over the extended source gives an answer of zero.

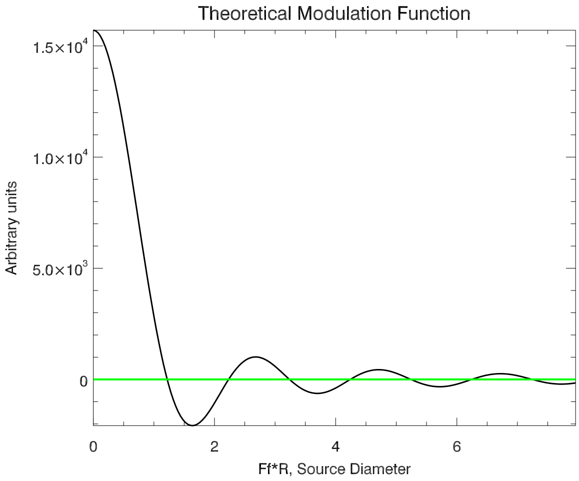

Fig. 8.— The numerically calculated Modulation Factor.

5.3. Finding exact hour-angle of zero crossing using Iterative Least Squares

Our discussion in section 5.2 alluded to the fact that we need to know the hour angle of the zero crossing of our interferometer response extremely accurately if we want to determine the radii of extended sources. This poses a challenge, since the zero point gets lost in the noise as the interferometer response goes to zero. To overcome the challenges of finding the zero point, we can use the techniques we learnt in finding the declination of a point source. Least Squares fitting comes to mind. We approximate the function we want to fit as the point source response modulated by a straight line - this is accurate as long as we stay in the region of the zero crossing. Thus, we want to fit the following as the fringe model:

$$F(ha) = [A\cos(2\pi f_f(h)) + B\sin(2\pi f_f(h))]\,(h - hz) \tag{22}$$

The obstacle is the non-linearity of the function we want to fit. Since the hour angle (or time, for that matter) that we want to calculate is the argument of trigonometric functions, we can't just do an easy least squares fit. Luckily, we can transform this into a differential problem and solve for *correction* values using least squares. We transform the problem of finding h_z through fitting Equation 22 into the problem of finding h_z through fitting:

$$F_{measured} - F_{guessed} = (\delta A\cos\alpha) + (\delta B\sin\alpha) + (-\delta h_z)[A\cos\alpha + B\sin\alpha] \tag{23}$$

where $\alpha = 2\pi f_f$ and F_i is the interferometer response. We start with guessed values of A, B and h_z. We use least squares to find δA, δB, δh_z, which is "correction" values we add to our guesses. If we start with a reasonable initial guess, this process converges to the true h_z zero crossing hour angle.

6. 1D PLOTTING OF THE SUN AND MOON

Since our interferometer has a fairly low angular resolution, our best extended sources are our very own sun and moon. We hope to observe them and derive their angular size - the only measurement possible for sources close to $\delta = 0$ with only two detectors like ours. The technique is applicable to any interferometry setup, though, whether we're at the VLA or on top of Campbell Hall. We use the techniques discussed in the previous section to find the hour angle of the zero crossings, find $f_f R$ and calculate angular diameter.

6.1. Background & Observations

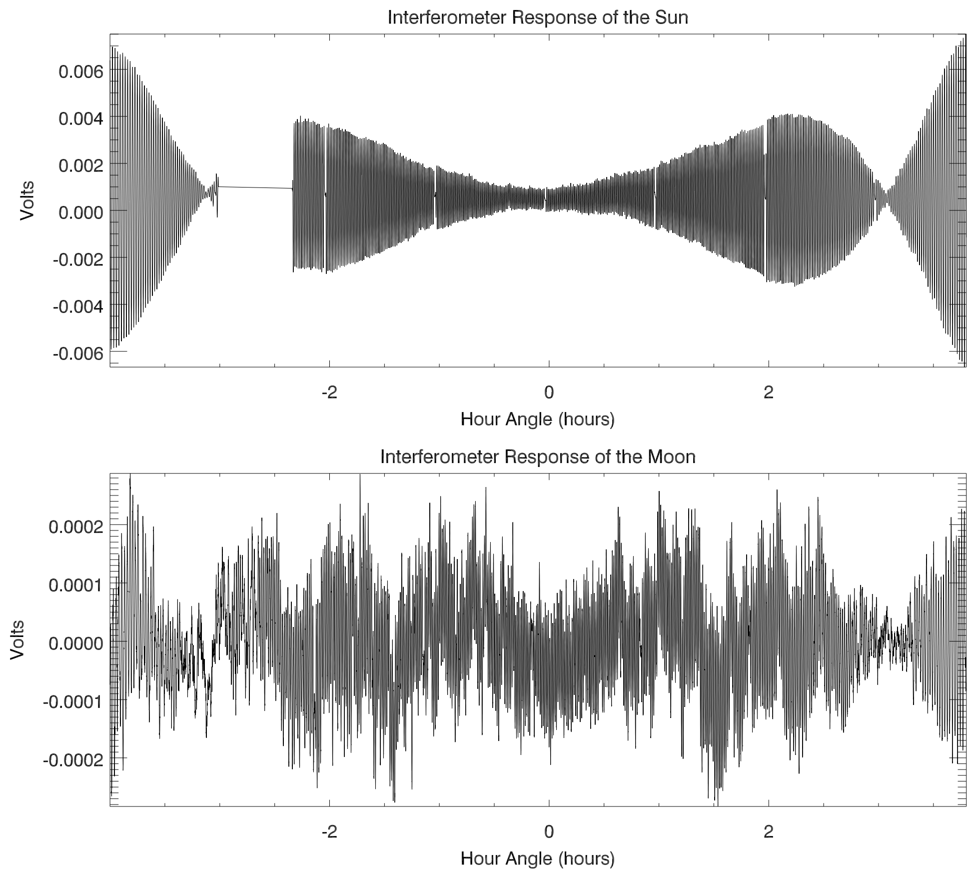

Fig. 9.— Interferometry Observation of the Sun and Moon.

The response to the sun looks exactly as we expect - a fringe frequency modulated by some Modulation Function. Notice the overall sausage-like shape of the interference pattern. This resembles the point source data modulated by another function dependent on hour angle. (*Due to an unfortunate series of technical difficulties, we lost some data around 1h, luckily this did not affect finding the zero crossings! Also, notice the tiny gaps in the data every hour - this is where dish calibration happened.*)

It is much harder to notice the sausage-like shape for the moon data, but the zero point around an hour angle of 3h is reasonably apparent. Also notice that a simple visual comparison shows that the zero-point seems to be at a very similar position to that of the sun's zero points. This agrees with our intuition - since the moon can eclipse the sun, the two has to be very similar in angular size.

6.2. Analysis

6.2.1. *Sun*

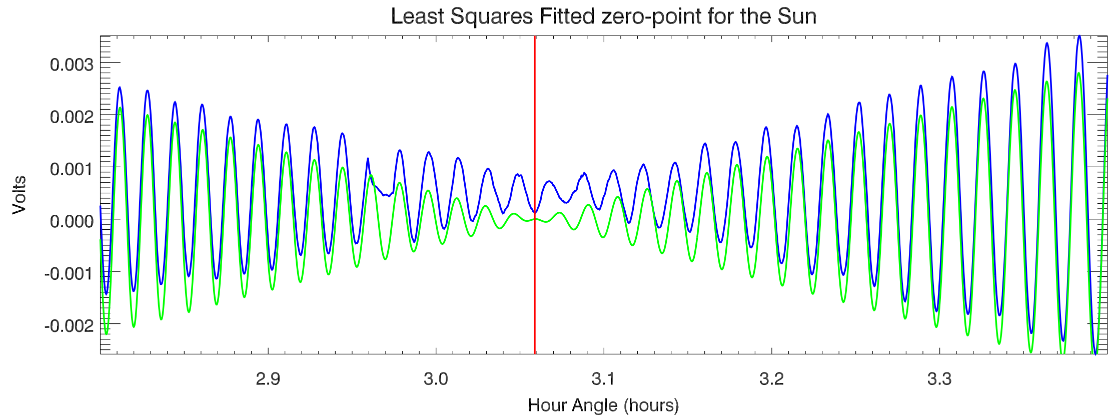

Fig. 10.— Fitting the fringe frequency modulated by a straight line to a small section of the sun data around the zero point to find the exact hour angle of the zero crossing. Notice how the shape of the original data (blue) coincides with the fitted plot (green). The final predicted zero crossing is indicated by the vertical line.

After applying the procedure described in section 5.3 to the raw sun data with an initial guess of h=3.2, we computed ha as 3.0587864 hours for the zero point of the sun fringe.

6.2.2. *Moon*

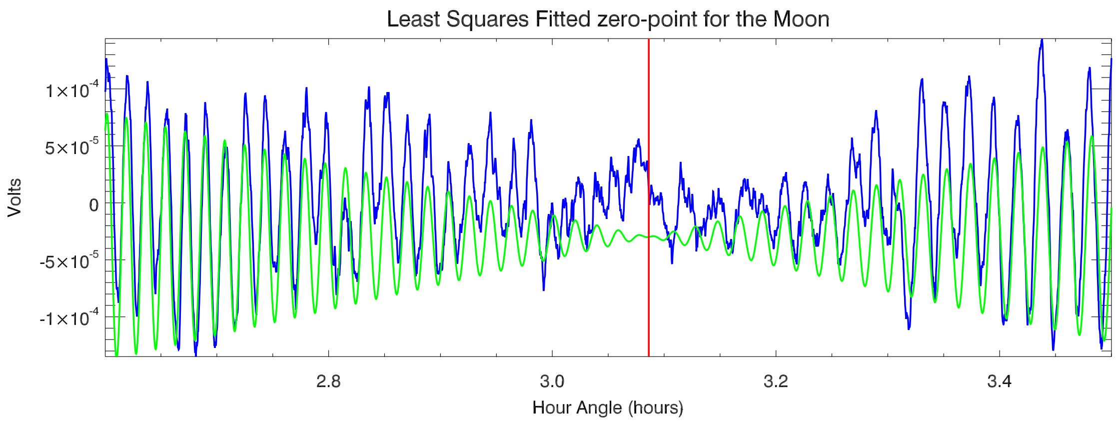

Fig. 11.— Fitting the fringe frequency modulated by a straight line to a small section of the moon data around the zero point to find the exact hour angle of the zero crossing. The fitted plot in green is artificially offset by -0.00003, notice how the shape coincides well with the original data (in blue). The final predicted zero crossing is indicated by the vertical line.

We apply the same procedure to the moon data after subtracting a two-degree poly-fit to normalize the data. We find ha to be 3.0862544 hours for the zero point of the moon fringe.

6.3. Results

We found hour angle values for the sun and the moon's zero crossings. We also compute the fringe frequency at the theoretical zero point that we get from analyzing equation 21 and figure 8.

6.3.1. Sun

We calculate the fringe frequency at the computed zero-crossing hour angle using formula 14 to find $f_f = 228.54\ cycles/radian$ We know the second zero occurs theoretically at $f_f R = 2.240$. Thus, we find our diameter to be:

$$\mathcal{D} = \frac{f_f R}{f_{data}} = \frac{2.24}{228.54} = 33.68\ arcminutes \tag{24}$$

6.3.2. Moon

We calculate the fringe frequency at the computed zero-crossing hour angle to find $f_f = 203.75\ cycles/radian$. Again, we know the second zero's theoretical position. Thus, we find the diameter to be:

$$\mathcal{D} = \frac{f_f R}{f_{data}} = \frac{2.24}{203.75} = 37.88\ arcminutes \tag{25}$$

This is slightly larger that expected, but it might be attributed to noisy data or computer arithmetic errors.

7. CONCLUSION

This lab covered much more ground than can be presented in only one report, but I believe I managed to cover all the fundamentals in this document. We started our foray into interferometry by applying our knowledge of heterodyne mixers to understand the signal flow path of an interferometer to enable the interference of received waves, thereby forming a fringe pattern. We covered the equatorial and horizontal coordinate system, where our most-used result was the relationship between hour angle and right ascension.

Once we could locate and observe sources we took a detailed look at radio interferometry. By using wave properties and mixing theory we derived the expected interferometer response to a monochromatic point source. This formula (Eq 13) formed the basis of interpreting and analyzing our data. First we used it to calculate accurate declination values using Least Squares fitting for the unknown A and B coefficients. We managed to find the declination of the Crab Nebula to within 10 arc seconds using this technique. We moved on to modeling the interferometry response to an extended source as a modulation function multiplied by the point source response. This model allowed us to calculate a theoretical modulation function for a round source. We used nonlinear least squares fitting to find accurate values for the hour angle of the sun and moon's fringe zero crossing. By combining the results from these two methods we manage to find the angular diameter of the sun.

We learnt new data analysis techniques, our programming proficiency increased dramatically, and we learnt to go without sleep for long periods of time, all crucial skills needed in the pursuit of science.

8. APPENDIX

8.1. Least Squares source code

```
pro leastsquares, X, Y, a, Ybar, s_sq, sigmaA, nCov
;+
;Takes the least-squares of the data, and calculates the errors
;
;ARGUMENTS
;X - The coefficient matrix
;Y - The data matrix
;
;RETURN VALUES
;a       - The derived coefficients
;Ybar    - The predicted data points
;sigmaA  - The uncertainties in the derived coefficients
;s_sq    - The sum of the squares of the residuals
;ncov    - The normalized covariance matrix
;-

M = size(X)
M = M[2]
N = size(X)
N = N[1]

XX = transpose(X)##X
XY = transpose(X)##Y
XXI = invert(XX)
a = XXI##XY

Ybar = X##a

deltaY = Y - Ybar
s_sq = transpose(deltaY)##deltaY
s_sq = s_sq/(M-N)
s_sq = s_sq[0]
diag = XXI[(N+1)*indgen(N)]
vardc = s_sq*diag
sigmaA = sqrt(vardc)
dc = XXI[(N+1)*indgen(N)]
ncov = XXI/sqrt(dc##dc)

end
```